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Domain-wall/Cosmology correspondence in adS/dS supergravity

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ABSTRACT

We realize the domain-wall/cosmology correspondence for (pseudo)supersymmetric domain walls (cosmologies) in the context of four-dimensional supergravity. The OSp(2|4)-invariant anti-de Sitter (adS) vacuum of a particular $N = 2$ Maxwell-Einstein supergravity theory is shown to correspond to the OSp($2^*|2,2$)-invariant de Sitter (dS) vacuum of a particular pseudo-supergravity model, with 'twisted' reality conditions on spinors. More generally, supersymmetric domain walls of the former model correspond to pseudo-supersymmetric cosmologies of the latter model, with time-dependent pseudo-Killing spinors that we give explicitly.

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1 Introduction

A scalar field minimally coupled to gravity is said to define a 'fake supergravity' theory [1] if the scalar potential V is given in terms of a triplet superpotential $\mathbf{W}$ by a certain 'supergravity-inspired' formula (see [2, 3] for related earlier work, and [4–7] for recent discussion of the multi-scalar, and other, generalizations). A domain-wall solution supported by the scalar field is then said to be (fake) supersymmetric if it admits a 'Killing spinor', defined as a non-zero solution for the complex doublet spinor field χ of the 'Killing spinor' equation

$$(D_\mu + \mathbf{W} \cdot \boldsymbol{\tau}\, \Gamma_\mu)\, \chi = 0\,, \qquad (\mu = 0, 1, \dots, D-1)\,, \tag{1.1}$$

where D_μ is the standard covariant derivative acting on Lorentz spinors, Γ_μ are Dirac matrices and $\boldsymbol{\tau}$ is a triplet of Pauli matrices. Remarkably, all domain walls that are either flat or 'adS-sliced' (foliated by anti-de Sitter spacetimes) are (fake) supersymmetric if the scalar field is strictly monotonic, because under these circumstances the required triplet superpotential can be constructed from the solution itself [1, 8–10]. The superpotential so constructed turns out to be real, and it takes the form $\mathbf{g}W$ for a flat wall, where $\mathbf{g}$ is a *fixed* 3-vector and W a real scalar superpotential.

It was shown in [9, 11] that similar results apply to flat and closed homogeneous and isotropic (Friedmann-Lemaitre-Robinson-Walker (FLRW)) cosmologies, despite their time-dependence. This result was found by an application of a 'domain-wall/cosmology (DW/C) correspondence', which states that for every maximally symmetric domain-wall solution of gravity coupled to scalar fields of a model with potential V there is a corresponding homogeneous and isotropic cosmology of the same model but with potential $-V$ [9, 12]. If the domain wall is adS-sliced then the corresponding cosmology is closed, and if the domain wall is flat then so is the corresponding cosmology. In either case, a domain-wall solution that is fake supersymmetric with respect to a (real) superpotential $\mathbf{W}$ corresponds to a cosmology that is fake supersymmetric with respect to the imaginary superpotential $i\mathbf{W}$. The corresponding solution of (1.1) was called a "pseudo-Killing" spinor in [9, 11] because the 'gamma-trace' of this equation is a Dirac-type equation but with an *anti-hermitian* 'mass' matrix. In this paper we will need only the special case of this result for flat domain walls and the corresponding flat cosmologies. Each fake supersymmetric flat domain wall is paired with a fake supersymmetric flat cosmology, and this pair is associated with some real scalar superpotential W such that there exist non-zero solutions χ of the equation

$$(D_\mu + W\, \mathbf{g} \cdot \boldsymbol{\tau}\, \Gamma_\mu)\, \chi = 0\,, \tag{1.2}$$

for some fixed 3-vector $\mathbf{g}$ that is real for the domain wall and imaginary for the cosmology.

Just as some fake supersymmetric domain-wall solutions of a fake supergravity theory may also be 'genuinely' supersymmetric solutions of 'genuine' supergravity theory (for a restricted set of possible spacetime dimensions D), one might expect some 'fake' pseudo-supersymmetric cosmological solutions to be 'genuinely' pseudo-supersymmetric solutions of some 'genuine' supergravity theory. However, Killing spinors arising in supergravity theories are generally subject to some reality (and/or chirality) condition. For example, for $D = 5$ the Killing spinor equation (1.1) can be deduced from the condition of vanishing supersymmetry variation of the gravitino field of $D = 5$ supergravity coupled to matter [4], and in this context the spinor χ is subject to a symplectic-Majorana condition that requires $\mathbf{W}$ to be real. Similar considerations apply in other dimensions and are expected to lead to the same conclusion (although the precise relation of the 'fake' Killing spinor equation (1.1) to the supergravity supersymmetry preservation conditions is known in only a few cases).

However, there exist 'non-standard' supergravity theories that are found by imposing 'twisted reality' conditions on spinors; we shall call them pseudo-supergravity theories. They first arose from an investigation of whether there could be supergravity theories with supersymmetric de Sitter (dS) vacua. The dS supergroups available as isometry supergroups were classified by Nahm [13] and are listed in table 1, along with the R-symmetry group. Applications for $D = 4, 5, 6$ have been discussed

Table 1: *de Sitter supergroups in $D \geq 4$ whose bosonic subgroup is* $\mathrm{SO}(D,1) \times R$.

	supergroup	R-symmetry	
$D = 6$	$\mathrm{F}^1(4)$		$\mathrm{SU}(2)$
$D = 5$	$\mathrm{SU}^*(4\|2n)$	$n = 1$	$\mathrm{SO}(1,1) \times \mathrm{SU}(2)$
		$n = 2$	$\mathrm{SO}(5,1)$
$D = 4$	$\mathrm{OSp}(N^*\|2,2)$	$N = 2$	$\mathrm{SO}(2)$
		$N = 4$	$\mathrm{SU}(1,1) \times \mathrm{SU}(2)$
		$N = 6$	$\mathrm{SU}(3,1)$
		$N = 8$	$\mathrm{SO}(6,2)$

in [14–18]; in particular, an explicit $N = 2$, $D = 4$ 'de Sitter' pseudo-supergravity was constructed in [15, 17]. Just as adS space can be viewed as a special case of a domain wall, so dS space can be viewed as a special case of an FLRW cosmology. This suggests that it should be possible to view pseudo-supergravity theories with supersymmetric dS vacua as 'duals' of 'standard' supergravity

theories with supersymmetric adS vacua, such that these two vacua are 'dual' in the sense of the DW/C correspondence. One purpose of this paper is to confirm this logic for a particular $N = 2$, $D = 4$, U(1)-gauged Maxwell-Einstein pseudo-supergravity theory that we show to be 'dual' in the sense just described to a 'standard' $N = 2$, $D = 4$ Maxwell-Einstein supergravity.

A further purpose of this paper is to extend this result to generic (pseudo)supersymmetric domain walls (cosmologies) of these 'dual' theories, although we shall limit ourselves here to solutions that are asymptotic to the adS (dS) vacuum. In either case the (pseudo)supersymmetry is shown to be a consequence of the existence of a non-zero solution for χ of (1.2) with $\mathbf{g} = (g, 0, 0)$, where g is the gauge coupling constant. While the standard reality conditions on χ imply that g is real, the twisted reality conditions imply that it is imaginary, exactly as required for pseudo-supersymmetry. When g is imaginary, reality of the action requires the U(1) gauge field to be imaginary too. This means that the kinetic term for a redefined, real, gauge field is negative, so the pseudo-supergravity has vector ghosts [15, 17]; this is a manifestation of the fact that there is no non-trivial representation of a dS superalgebra in a positive definite Hilbert space.

This paper is organized as follows. In the next section we recall the essentials of $N = 2$, $D = 4$ supergravity and the different reality conditions that one may impose on the spinors. In particular, for a choice of gauging we show that standard reality conditions lead to a supersymmetric adS critical point while twisted reality conditions to a supersymmetric dS critical point. We further relate this to fake supergravity. In section 3 we realize the domain-wall/cosmology correspondence in supergravity by finding the corresponding supersymmetric domain-wall/cosmology solutions. Section 4 contains our conclusions and further discusses some implications.

We became aware of related work on a realization of the DW/C correspondence in $D = 10$ and $D = 11$ supergravity [19] during the completion of an earlier version of this paper. This revision presents the precise relation of our supergravity results to the fake supergravity formalism in which context the correspondence was originally proposed.

2 $N = 2$ gauged (pseudo)supergravity

In this section we review the features of $D = 4$, $N = 2$ Einstein-Maxwell supergravity theory [20] relevant for our application, in particular the choices of reality conditions on spinors, and we also show how the (pseudo)Killing spinor equation (1.2) arises in this context. For one vector multiplet, the bosonic fields are the metric $G_{\mu\nu}$, two vectors A^I_μ (I=0, 1), (with A^0_μ being the graviphoton) and a complex scalar z; the fermionic fields are the two gravitini ψ^i_μ and the two photini λ^i, ($i = 1, 2$).

A linear combination $g_I A^I_\mu$ of the two vector fields may be used to gauge a U(1) group. We will consider a model with an SO(1,1)-invariant metric on the space of coupling vectors g_I, leading to three types of gauging according to whether this vector is 'timelike', 'spacelike' or 'null'. For the most part we follow the notations and conventions in [21].

2.1 $N = 2$ supergravity with one vector multiplet

As we are interested in domain-wall/cosmology solutions that involve only the metric and the scalar fields, we truncate the supergravity theory to this sector. The truncated Lagrangian density $\mathcal{L}$ is given by

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - g_{z\bar z} D_\mu z D^\mu \bar z \;-\; V(z,\bar z)\,, \tag{2.1}$$

where $e = \sqrt{-\det G}$, and $g_{z\bar z}$ is the Kähler target space metric, given in terms of a Kähler potential $\mathcal{K}$ by

$$g_{z\bar z} = \partial_z \partial_{\bar z}\mathcal{K}\,. \tag{2.2}$$

The relations of special geometry imply that

$$\mathrm{e}^{-\mathcal{K}} = -\mathrm{i}\left(Z^I \frac{\partial \bar F}{\partial \bar Z^I} - \bar Z^I \frac{\partial F}{\partial Z^I}\right)\,, \tag{2.3}$$

where $Z^I(z)$ $(I = 0, 1)$ are two holomorphic functions of z, and $F(Z)$ is a holomorphic function of these two variables, homogeneous of second degree. Due to the homogeneity, one of the variables Z^I is irrelevant, so one can take an arbitrary parametrization (up to some requirements of non-degeneracy) of the Z^I in terms of z.

The form of the potential V is determined, via a Ward identity, from the supersymmetry transformations rules of the gravitini and photini [22, 24, 25], so we consider these first. We take the spinor parameters to be ϵ^i $(i = 1, 2)$ and we work with chiral spinors, i.e. eigenspinors of $\gamma_5 \equiv \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$, with the position of the index indicating chirality:

$$\begin{aligned}
&\epsilon^i = +\gamma_5 \epsilon^i\,, \qquad {\psi_\mu}^i = +\gamma_5 {\psi_\mu}^i\,, \qquad \lambda^i = -\gamma_5 \lambda^i\,,\\
&\epsilon_i = -\gamma_5 \epsilon_i\,, \qquad \psi_{\mu i} = -\gamma_5 \psi_{\mu i}\,, \qquad \lambda_i = +\gamma_5 \lambda_i\,.
\end{aligned} \tag{2.4}$$

After truncation to the metric-scalar sector, the fermion field supersymmetry transformation laws are

$$\begin{aligned}
\delta\psi_{\mu i} &= \left(D_\mu - \tfrac{1}{2}\mathrm{i}A_\mu\right)\epsilon_i - \gamma_\mu S_{ij}\epsilon^j\,, \qquad \delta\lambda^i = \partial\!\!\!/ \bar z \epsilon^i + N^{\bar z\, ij}\epsilon_j\,,\\
\delta\psi^i_\mu &= \left(D_\mu + \tfrac{1}{2}\mathrm{i}A_\mu\right)\epsilon^i - \gamma_\mu S^{ij}\epsilon_j\,, \qquad \delta\lambda_i = \partial\!\!\!/ z \epsilon_i + N^z_{ij}\epsilon^j\,,
\end{aligned} \tag{2.5}$$

where D_μ is the usual Lorentz-covariant derivative on spinors, and

$$A_\mu = -\tfrac{1}{2}\mathrm{i}\left(\partial_\mu z\,\partial_z\mathcal{K} - \partial_\mu\bar z\,\partial_{\bar z}\mathcal{K}\right) \tag{2.6}$$

is the Kähler connection. The auxiliary fields are given by

$$\begin{aligned} S_{ij} &= -P_{Iij}\,\mathrm{e}^{\mathcal{K}/2}\bar Z^I\,, \qquad N^{\bar z\,ij} = -2\mathrm{e}^{\mathcal{K}/2}P_I^{ij}\,g^{z\bar z}\mathcal{D}_z Z^I\,,\\ S^{ij} &= -P_I^{ij}\mathrm{e}^{\mathcal{K}/2}Z^I\,, \qquad N^z_{ij} = -2\mathrm{e}^{\mathcal{K}/2}P_{Iij}\,g^{z\bar z}\,\overline{\mathcal{D}_z Z^I}\,, \end{aligned} \tag{2.7}$$

where $g^{z\bar z} = (g_{z\bar z})^{-1}$, and

$$\mathcal{D}_z Z^I = \partial_z Z^I + Z^I\partial_z\mathcal{K} \tag{2.8}$$

is the Kähler-covariant derivative of Z^I. One also has

$$P_I^{ij} = \varepsilon^{ik}\varepsilon^{j\ell}P_{Ik\ell}\,, \tag{2.9}$$

where P_{Iij} (the moment map) will be specified below. We use conventions for which $\varepsilon^{ij}\varepsilon_{kj} = \delta^i_k$ and $\varepsilon_{12} = \varepsilon^{12} = 1$. As mentioned, the scalar potential follows directly from the transformation laws, and is given by

$$V = -6S^{ij}S_{ij} + \tfrac{1}{2}g_{z\bar z}N^z_{ij}N^{\bar z ij}\,. \tag{2.10}$$

In the absence of physical hypermultiplets, P_{Iij} are the entries of two constant symmetric matrices P_I, and an 'equivariance condition' requires these matrices to be proportional, so

$$P_{Iij} = g_I e_{ij}\,, \tag{2.11}$$

for constants g_I (which are the components of the coupling constant vector mentioned earlier) and constant symmetric matrix e_{ij}. Introducing a triplet of Pauli matrices τ, we may write

$$e_{ij} = \left[\tau_2\left(\mathbf{n}\cdot\tau\right)\right]_{ij}\,, \tag{2.12}$$

for 3-vector $\mathbf{n} = (n^1, n^2, n^3)$, which is complex, a priori, but will be restricted by reality conditions to be discussed below. The supersymmetry transformations (2.5) may now be written as

$$\begin{aligned} \delta\psi_{\mu i} &= \left(D_\mu - \tfrac{1}{2}\mathrm{i}A_\mu\right)\epsilon_i + \mathrm{e}^{\mathcal{K}/2}\bar{\mathcal{Z}}\gamma_\mu\left[\tau_2\left(\mathbf{n}\cdot\boldsymbol{\tau}\right)\right]_{ij}\epsilon^j\,,\\ \delta\psi_\mu^i &= \left(D_\mu + \tfrac{1}{2}\mathrm{i}A_\mu\right)\epsilon^i + \mathrm{e}^{\mathcal{K}/2}\mathcal{Z}\gamma_\mu\left[\left(\mathbf{n}\cdot\boldsymbol{\tau}\right)\tau_2\right]^{ij}\epsilon_j\,,\\ \delta\lambda^i &= \not{\partial}\bar z\,\epsilon^i - 2\mathrm{e}^{\mathcal{K}/2}g^{z\bar z}\,\mathcal{D}_z\mathcal{Z}\left[\left(\mathbf{n}\cdot\boldsymbol{\tau}\right)\tau_2\right]^{ij}\epsilon_j\,,\\ \delta\lambda_i &= \not{\partial}z\,\epsilon_i - 2\mathrm{e}^{\mathcal{K}/2}g^{z\bar z}\,\overline{\mathcal{D}_z\mathcal{Z}}\left[\tau_2\left(\mathbf{n}\cdot\boldsymbol{\tau}\right)\right]_{ij}\epsilon^j\,, \end{aligned} \tag{2.13}$$

where

$$\mathcal{Z} = g_I Z^I\,, \qquad \mathcal{D}_z\mathcal{Z} = \partial_z\mathcal{Z} + \mathcal{Z}\partial_z\mathcal{K}\,,$$
$$\bar{\mathcal{Z}} = g_I \bar{Z}^I\,, \qquad \overline{\mathcal{D}_z\mathcal{Z}} = \partial_{\bar{z}}\bar{\mathcal{Z}} + \bar{\mathcal{Z}}\partial_{\bar{z}}\mathcal{K}\,. \tag{2.14}$$

The potential may similarly be written as

$$V = 4\,(\mathbf{n}\cdot\mathbf{n})\,\mathrm{e}^{\mathcal{K}}\left[g^{z\bar{z}}\mathcal{D}_z\mathcal{Z}\,\overline{\mathcal{D}_z\mathcal{Z}} - 3\mathcal{Z}\bar{\mathcal{Z}}\right]\,. \tag{2.15}$$

2.2 Kähler-gauge invariant formulation

This potential (2.15) is invariant under the Kähler gauge transformations

$$\mathcal{K} \to \mathcal{K} - \left(f + \bar{f}\right)\,, \qquad Z^I \to \mathrm{e}^f Z^I\,, \qquad \bar{Z}^I \to \mathrm{e}^{\bar{f}}\bar{Z}^I\,, \tag{2.16}$$

which induces the transformations

$$A_\mu \to A_\mu + \tfrac{1}{2}\mathrm{i}\partial_\mu\left(f - \bar{f}\right)\,, \qquad \mathcal{Z} \to \mathrm{e}^f\mathcal{Z}\,, \qquad \bar{\mathcal{Z}} \to \mathrm{e}^{\bar{f}}\bar{\mathcal{Z}}\,. \tag{2.17}$$

This suggests that we introduce the new, gauge-equivalent, Kähler potential

$$\tilde{\mathcal{K}} = \mathcal{K} + \log\left(\mathcal{Z}\bar{\mathcal{Z}}\right)\,, \tag{2.18}$$

and its associated, gauge-equivalent, Kähler connection,

$$\tilde{A}_\mu = A_\mu - \tfrac{1}{2}\mathrm{i}\partial_\mu\log\left(\mathcal{Z}/\bar{\mathcal{Z}}\right)\,. \tag{2.19}$$

In terms of the function [23]

$$W = \mathrm{e}^{\tilde{\mathcal{K}}/2}\,, \tag{2.20}$$

the scalar potential takes the manifestly Kähler-gauge invariant form

$$V = 16\,(\mathbf{n}\cdot\mathbf{n})\left[g^{z\bar{z}}\partial_z W\,\partial_{\bar{z}}W - \frac{3}{4}W^2\right]\,. \tag{2.21}$$

The supersymmetry transformation laws may be similarly written in Kähler-gauge invariant form by introducing the new spinor parameters

$$\tilde{\epsilon}_i = \left(\mathcal{Z}/\bar{\mathcal{Z}}\right)^{\frac{1}{4}}\epsilon_i\,, \qquad \tilde{\epsilon}^i = \left(\bar{\mathcal{Z}}/\mathcal{Z}\right)^{\frac{1}{4}}\epsilon^i\,. \tag{2.22}$$

One then finds that

$$\begin{aligned}
\left(\mathcal{Z}/\bar{\mathcal{Z}}\right)^{\frac{1}{4}}\delta\psi_{\mu i} &= \left(D_\mu - \tfrac{1}{2}\mathrm{i}\tilde{A}_\mu\right)\tilde{\epsilon}_i + W\gamma_\mu\left[\tau_2\,(\mathbf{n}\cdot\boldsymbol{\tau})\right]_{ij}\tilde{\epsilon}^j\,,\\
\left(\bar{\mathcal{Z}}/\mathcal{Z}\right)^{\frac{1}{4}}\delta\psi_\mu^i &= \left(D_\mu + \tfrac{1}{2}\mathrm{i}\tilde{A}_\mu\right)\tilde{\epsilon}^i + W\gamma_\mu\left[(\mathbf{n}\cdot\boldsymbol{\tau})\,\tau_2\right]^{ij}\tilde{\epsilon}_j\,,\\
\left(\bar{\mathcal{Z}}/\mathcal{Z}\right)^{\frac{1}{4}}\delta\lambda^i &= \partial\!\!\!/\bar{z}\,\tilde{\epsilon}^i - 4g^{z\bar{z}}\partial_z W\left[(\mathbf{n}\cdot\boldsymbol{\tau})\,\tau_2\right]^{ij}\tilde{\epsilon}_j\,,\\
\left(\mathcal{Z}/\bar{\mathcal{Z}}\right)^{\frac{1}{4}}\delta\lambda_i &= \partial\!\!\!/z\,\tilde{\epsilon}_i - 4g^{z\bar{z}}\partial_{\bar{z}}W\left[\tau_2\,(\mathbf{n}\cdot\boldsymbol{\tau})\right]_{ij}\tilde{\epsilon}^j\,.
\end{aligned} \tag{2.23}$$

2.3 The model

We will choose the prepotential

$$F(Z) = \frac{\mathrm{i}}{4}(-Z^0 Z^0 + Z^1 Z^1)\,, \tag{2.24}$$

which yields

$$\mathrm{e}^{-\mathcal{K}(z,\bar{z})} = Z^0 \bar{Z}^0 - Z^1 \bar{Z}^1\,. \tag{2.25}$$

Because of the homogeneity of F, we may choose $Z^0 = 1$, and we may then choose a parametrization such that $Z^1 = z$. Thus, without loss of generality we may choose

$$Z^I = (1, z)\,. \tag{2.26}$$

This yields

$$\mathcal{K}(z,\bar{z}) = -\log\left(1 - z\bar{z}\right)\,, \qquad \mathcal{Z} = g_0 + g_1 z\,, \tag{2.27}$$

and hence

$$g_{z\bar{z}} = (1 - z\bar{z})^{-2}\,, \qquad A_\mu = -\tfrac{1}{2}\mathrm{i}\,(1 - z\bar{z})^{-1}\left[\bar{z}\partial_\mu z - z\partial_\mu \bar{z}\right]\,, \tag{2.28}$$

and

$$W^2 = \frac{(g_0 + g_1 z)\,(g_0 + g_1 \bar{z})}{1 - z\bar{z}}\,. \tag{2.29}$$

The values of the complex scalar field z must be restricted to the unit disc $z\bar{z} < 1$, and the metric is then the SU(1,1)-invariant hyperbolic metric on this disc. From the formula (2.21) we find that

$$V = \frac{4\,(\mathbf{n}\cdot\mathbf{n})}{1 - z\bar{z}}\,\left[g_0^2\,(z\bar{z} - 3) - 2g_0 g_1\,(z + \bar{z}) + g_1^2\,(1 - 3z\bar{z})\right]\,. \tag{2.30}$$

When $g_0 = g_1$ there is no extremum of V within the unit disc, $z\bar{z} < 1$. Otherwise there is an extremum, which is at $z = 0$ for the two special cases in which either $g_0 = 0$ or $g_1 = 0$.

As we will see in the section to follow, standard reality conditions imply that $\mathbf{n}$ is a real 3-vector, in which case $V > 0$ when $g_0 = 0$ (but $g_1 \neq 0$), and the minimum of V at $z = 0$ is a supersymmetry breaking dS vacuum. In contrast, $V < 0$ when $g_1 = 0$ (but $g_0 \neq 0$), and the maximum at $z = 0$ is a supersymmetric adS vacuum. This is the case that we will focus on in this paper. For our purposes, it will suffice to consider $g_I = (1, 0)$, so that

$$\mathcal{Z} = 1\,, \qquad W = 1/\sqrt{1 - z\bar{z}}\,. \tag{2.31}$$

The scalar potential for this model is

$$V = 4\,(\mathbf{n}\cdot\mathbf{n})\left[\frac{z\bar{z} - 3}{1 - z\bar{z}}\right]\,. \tag{2.32}$$

For $\mathbf{n}$ a real 3-vector, this is precisely the potential of the SO(4) gauged $N = 4$ supergravity, which has an identical scalar field content. It follows that we are considering an $N = 2$ truncation of this $N = 4$ model, and hence of $SO(8)$ gauged $N = 8$ supergravity, as also follows from results of [26] on the $\mathrm{U}(1)^4$ truncation of the $N = 8$ theory[1]. As the $N = 8$ theory is a consistent truncation of the S^7-compactification of $D = 11$ supergravity, any domain-wall solution of our model, such as the one found later, can be lifted to $D = 11$, using e.g. the results of [27, 28].

Our next task is to consider the implications for this model of 'twisted' reality conditions on the fermion fields.

2.4 Reality conditions

We use conventions for which the charge conjugation matrix is γ^0, so that all Dirac matrices γ_μ are real and $\gamma_5 = \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$ is imaginary. This implies that complex conjugation changes the chirality of the spinors. In our conventions, complex conjugation does not change the order of spinors, so $(\lambda\chi)^* = \lambda^*\chi^*$. Although chiral spinors are necessarily complex, each component of a chiral spinor must be linearly related to the complex conjugate of the corresponding anti-chiral spinor in order that each spinor has 4 independent complex components as required by $N = 2$ supersymmetry. In other words, we require a 'reality condition' of the form [15, 17, 29]

$$\epsilon^i = M^i{}_j\,(\epsilon_j)^*\;, \tag{2.33}$$

and similarly for other spinors, where $M^i{}_j$ are the entries of an invertible matrix M that must be hermitian for reality of the action. In particular, the redefined spinor parameters $\tilde{\epsilon}^i$ and $\tilde{\epsilon}_i$ introduced in (2.22) will satisfy exactly the same reality condition as the spinors ϵ^i and ϵ_i. By a redefinition of the spinors one can send

$$M \to S^\dagger M\,S\,, \tag{2.34}$$

where S is any invertible matrix, and one may choose S so as to bring M to one of two standard forms: $M = 1$ or $M = \mathbf{m}\cdot\boldsymbol{\tau}$ for some real unit 3-vector $\mathbf{m}$. The choice $M = 1$ leads to standard $N = 2$ supergravity while the choice $M = \mathbf{m}\cdot\boldsymbol{\tau}$ (e.g. $M = \tau_3$) leads to $N = 2$ pseudo-supergravity. In the former case the spinors are SU(2) doublets and in the latter case they are $\mathrm{SU}(1,1)$ doublets.

[1] This involves keeping only the 35 scalars parametrizing $\mathrm{S}\,\ell(8;R)/\,\mathrm{SO}(8)$, as a first step, then using the local SO(8) invariance to diagonalize the $\mathrm{S}\,\ell(8;R)$ matrix, and retaining only the 8 diagonal entries X_α ($\alpha = 1,\ldots,8$), as a second step. The potential depends only on the 7 scalars parametrizing the subspace defined by $\prod_\alpha X_\alpha = 1$, and a further truncation obtained by choosing the particular solution $X_1 = X_2 = X_3 = X_4 = X$ and $X_5 = X_6 = X_7 = X_8 = X^{-1}$ yields our model with $X = \mathrm{e}^{\sigma/2}$.

Consistency of the 'reality condition' (2.33) with the fermionic field supersymmetry variations of (2.13) requires (for g_I real; we henceforth restrict to $g_0 = 1$ and $g_1 = 0$)

$$(\mathbf{n} \cdot \tau)\, \tau_2 = M\, (\mathbf{n}^* \cdot \tau)\, \tau_2 M^* \,. \tag{2.35}$$

The fermion field variations of (2.23) may now be written, suppressing SU(2) or SU(1,1) indices, in terms of a complex chiral doublet photino field λ and a complex anti-chiral doublet gravitino field ψ_μ. Recalling that $\mathcal{Z} = 1$ for the model of interest here, we have

$$\begin{aligned} \delta\psi_\mu &= \left(D_\mu - \tfrac{1}{2}\mathrm{i}A_\mu\right)\epsilon + W\gamma_\mu\tau_2\,(\boldsymbol{\tau}\cdot\mathbf{n})\,M\epsilon^* \,, \\ \delta\lambda &= \not{\partial} z\,\epsilon - 4g^{z\bar{z}}\partial_{\bar{z}}W\,\tau_2\,(\mathbf{n}\cdot\boldsymbol{\tau})\,M\epsilon^* \,, \end{aligned} \tag{2.36}$$

where ϵ is a complex anti-chiral doublet spinor parameter.

The consistency condition (2.35) should be viewed as a reality condition on the 3-vector $\mathbf{n}$. For $M = 1$ it implies that $\mathbf{n}$ is real, whereas for $M = \mathbf{m}\cdot\boldsymbol{\tau}$, it implies that the components of $\mathbf{n}$ perpendicular to $\mathbf{m}$ are real but the component (anti)parallel to $\mathbf{m}$ is imaginary. In either case we may write

$$\mathbf{n} = g\,\mathbf{m} + \mathbf{n}_\perp \,, \qquad \mathbf{n}_\perp\cdot\mathbf{m} = 0 \,, \qquad \mathbf{n}_\perp,\, \mathbf{m} \in \mathbb{R}^3 \,, \tag{2.37}$$

where g is real[2] for $M = 1$ (in which case $\mathbf{m}$ should be interpreted as an arbitrary unit 3-vector) but imaginary [17] for $M = \mathbf{m}\cdot\boldsymbol{\tau}$. In the former case we have $V < 0$ and the potential has a supersymmetric adS maximum at $z = 0$. In the latter case, the potential may be positive, negative or zero, depending on the relative magnitudes of g and $\mathbf{n}_\perp$. In particular, when $g = 0$, but $\mathbf{n}_\perp \neq \mathbf{0}$, we again have $V < 0$ with a supersymmetric adS vacuum at $z = 0$, but the isometry supergroup is OSp$(1,1|4)$ rather than OSp$(2|4)$. When $\mathbf{n}_\perp = \mathbf{0}$ but $g \neq 0$ we have $V > 0$ and what was the supersymmetric adS vacuum is now a supersymmetric dS vacuum, with isometry supergroup OSp$(2^*|2,2)$. The various possible supersymmetric (a)dS vacua, along with their isometry supergroups (and bosonic subgroups) are shown in table 2.

Henceforth, we make the standard choice for twisted reality conditions: $M = \tau_3$, corresponding to $\mathbf{m} = (0,0,1)$. We will also choose

$$\mathbf{n} = g\,\mathbf{m} = g\,(0,0,1)\,. \tag{2.38}$$

This implies no loss of generality when $M = 1$ but amounts to the choice $\mathbf{n}_\perp = 0$ for twisted reality

[2] This is the coupling constant mentioned in the Introduction; it can be viewed as the U(1) gauge coupling constant because it follows from (2.11) that the vector coupling constant is really gg_I and we have set $g_I = (1,0)$.

Table 2: *(Pseudo)supersymmetric (a)dS vacua for $D = 4$, $N = 2$ supergravity minimally coupled to one vector multiplet.*

Supergravity	$\mathbf{n} \perp \mathbf{m}$	$\mathbf{n} \parallel \mathbf{m}$
Normal	adS $\times$ SO(2)	
$M = 1$	OSp(2|4)	
Pseudo	adS $\times$ SO(1,1)	dS $\times$ SO(2)
$M = \mathbf{m} \cdot \boldsymbol{\tau}$	OSp(1,1|4)	OSp(2*|2,2)

conditions. In either case the potential (2.32) becomes

$$V = -4g^2 \left[\frac{3 - |z|^2}{1 - |z|^2} \right] , \tag{2.39}$$

but g is real for $M = 1$ and imaginary for $M = \tau_3$.

Our interest in the supersymmetry transformation laws is primarily due to the fact that one gets the conditions for preservation of supersymmetry, in a bosonic background, by setting to zero the supersymmetry variations of the fermion fields. So, for simplicity, we now set $\delta\psi_\mu$ and $\delta\lambda$ to zero in (2.36) to arrive at the supersymmetry preservation conditions

$$\begin{aligned} 0 &= \left(D_\mu - \tfrac{1}{2}\mathrm{i}A_\mu\right)\epsilon + \mathrm{i}g\, W \gamma_\mu \tau_1 M \epsilon^* , \\ 0 &= \partial\!\!\!/ z\, \epsilon - 4\mathrm{i}g\, g^{z\bar{z}} \partial_{\bar{z}} W \tau_1 M \epsilon^* , \end{aligned} \tag{2.40}$$

where $W = 1/\sqrt{1 - |z|^2}$ and either $M = 1$ or $M = \tau_3$, with g real for $M = 1$ and imaginary for $M = \tau_3$.

2.5 Relation to fake (pseudo)supergravity

For the models discussed in the previous section, the potential $V(z, \bar{z})$ actually depends only on $|z|$. This suggests that we write

$$z = \rho(\sigma) e^{\mathrm{i}\phi} , \qquad \rho(\sigma) = \tanh(\sigma/2) , \tag{2.41}$$

for real fields σ and ϕ. In terms of these new fields, the Lagrangian density is given by

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}\left[(\partial\sigma)^2 + \sinh^2\sigma\,(\partial\phi)^2\right] - V . \tag{2.42}$$

Moreover, we have

$$W = \cosh(\sigma/2) \qquad W' \equiv \partial_\sigma W = \frac{1}{2}\sinh(\sigma/2) , \tag{2.43}$$

and the potential is

$$V = 16g^2 \left[\left(W'\right)^2 - \frac{3}{4} W^2 \right] = -4g^2 \left(2 + \cosh\sigma\right) \,. \tag{2.44}$$

The dependence of V on W is precisely that of fake (pseudo)supergravity.

The supersymmetry preservation conditions (2.40) may be simplified by writing the anti-chiral doublet of spinors ϵ as

$$\epsilon = \frac{1}{2} \left(1 - \gamma_5\right) \chi \,, \tag{2.45}$$

where χ is a doublet of spinors satisfying the reality condition[3]

$$\chi^* = M\chi \,. \tag{2.46}$$

Passing to the new target space coordinates, we find that equations (2.40) become

$$\begin{aligned} 0 &= \left[D_\mu + \tfrac{1}{2}\mathrm{i}\gamma_5 A_\mu + gW\Gamma_\mu \tau_1\right] \chi \,, \\ 0 &= \left[\Gamma^\mu \left(\partial_\mu \sigma + \mathrm{i}\gamma_5 \sinh\sigma \, \partial_\mu \phi\right) - 8gW'\tau_1\right] \chi \,, \end{aligned} \tag{2.47}$$

where

$$A_\mu = \sinh^2\left(\sigma/2\right) \partial_\mu \phi \,, \tag{2.48}$$

and

$$\Gamma_\mu = \mathrm{i}\gamma_\mu \gamma_5 \,, \tag{2.49}$$

are alternative Dirac matrices[4].

The first of equations (2.47) is the Killing spinor equation; note that the field ϕ enters only through the Kähler connection A_μ, which is proportional to $\partial_\mu \phi$. For any configuration that depends on a single coordinate, such as the domain wall and cosmological configurations of interest, the Kähler field strength (which is the pullback of the Kähler 2-form) will vanish. This means that the term involving ϕ in the Killing spinor equation is irrelevant to the integrability conditions of this equation, which are the same as those of the simpler equation

$$\left(D_\mu + g\,\Gamma_\mu W \tau_1\right) \chi = 0 \,. \tag{2.50}$$

This equation is precisely of the fake-supergravity form (1.2) with $\mathbf{g} = (g, 0, 0)$. For a domain-wall or cosmology background of the type to be considered here, it is known [1, 8, 9] that this Killing spinor equation implies that

$$\left[\Gamma^\mu \partial_\mu \sigma - 8g\, W'\, \tau_1\right] \chi = 0 \,. \tag{2.51}$$

[3] This condition is consistent since $MM^* = 1$ for both $M = 1$ and $M = \tau_3$.

[4] The matrices $\Gamma^0\Gamma_\mu$ are real and symmetric, so one may choose $\Gamma^0 = \mathrm{i}\gamma^0\gamma_5$ as the charge conjugation matrix in this representation, replacing the choice γ^0 in the representation γ_μ.

This agrees with the second of equations (2.47) if and only if ϕ is constant, and it then follows that A_μ vanishes, so that (pseudo)Killing spinors are in fact solutions of (2.50).

To summarize, we have shown that necessary and sufficient conditions for a domain wall (cosmology) to be a supersymmetric solution of our (pseudo)supergravity model are (i) that z have constant phase and (ii) that (2.50) admit a non-zero solution for χ.

3 Domain-wall/Cosmology correspondence

The metric for a $D = 4$ flat domain-wall spacetime may be put in the standard form

$$ds^2 = dr^2 + \mathrm{e}^{2A(r)} ds^2(\mathrm{Mink3})\,, \tag{3.1}$$

where r represents distance in a direction perpendicular to the wall, so that the geometry is determined by the function $A(r)$, and Mink3 is the 3-dimensional Minkowski metric. The generic isometries of this metric are those of the $D = 3$ Poincaré group, and to preserve this symmetry we must take the scalar fields to depend only on r. Given a solution of this form for a model with scalar field potential V, the DW/C correspondence states that the same model but with scalar field potential $-V$ has a cosmological solution with metric

$$ds^2 = -dt^2 + \mathrm{e}^{2A(t)} dl^2(\mathrm{E3}) \tag{3.2}$$

and scalar fields that depend only on time, where E3 is a 3-dimensional Euclidean metric. This is of standard FLRW form with $A(t)$ being the logarithm of the scale factor.

We have seen that (pseudo)supersymmetric solutions of our model are such that the complex field z has constant phase ϕ. The $\phi = 0$ truncation of (2.42) is clearly consistent, so we effectively have a model with a single scalar field σ. From the results of [9], we then learn that (pseudo)supersymmetric domain walls (cosmologies) are such that (in current notation and conventions)

$$\begin{aligned} \dot{A} &= \pm 2|g|W = \pm 2|g|\cosh(\sigma/2)\,, \\ \dot{\sigma} &= \mp 8|g|W' = \mp 4|g|\sinh(\sigma/2)\,, \end{aligned} \tag{3.3}$$

where W is the scalar superpotential appearing in (2.50)-(2.51), given by (2.43) for the model in hand, and the overdot indicates differentiation with respect to the independent variable, which is the distance variable r in the domain-wall case and the time variable t in the cosmology case.

3.1 Domain walls

As a check of equations (3.3) for the domain-wall case let us return to (2.47). The coupling constant g is real, and we may assume it to be positive without loss of generality. If one assumes[5] that χ depends only on r then the projection of the first of eqs. (2.47) in any direction parallel to the wall yields

$$\left(\dot{A} + 2gW\Gamma\right)\chi = 0\,, \qquad \Gamma \equiv \Gamma_r \tau_1\,, \tag{3.4}$$

where the first term comes from the spin connection; note that there is no contribution from the Kähler connection because its only non-zero component is A_r (as a consequence of the fact that z is a function only of r). Writing χ as the sum of eigenspinors of Γ,

$$\chi = \chi_+ + \chi_-\,, \qquad \Gamma\chi_\pm = \pm\chi_\pm\,, \tag{3.5}$$

we see that

$$\left(\dot{A} + 2gW\right)\chi_+ + \left(\dot{A} - 2gW\right)\chi_- = 0\,. \tag{3.6}$$

Acting on this equation with Γ yields

$$\left(\dot{A} + 2gW\right)\chi_+ - \left(\dot{A} - 2gW\right)\chi_- = 0\,, \tag{3.7}$$

and hence

$$\left(\dot{A} \pm 2gW\right)\chi_\pm = 0\,, \tag{3.8}$$

for either choice of sign. Given $gW \neq 0$ and $\chi \neq 0$, it follows that

$$\dot{A} = \pm 2gW\,, \qquad \chi = \chi_\mp\,, \tag{3.9}$$

for *either* the top sign *or* the bottom sign. Given this restriction on χ, the second of equations (2.47) in a domain-wall background becomes

$$\left(\dot{\sigma} + \mathrm{i}\gamma_5 \sinh\sigma\,\dot{\phi} \pm 8gW'\right)\chi_\mp = 0\,. \tag{3.10}$$

Multiplying this equation by Γ and using the fact that Γ anticommutes with γ_5, we get the equation

$$\left(\dot{\sigma} - \mathrm{i}\gamma_5 \sinh\sigma\,\dot{\phi} \pm 8gW'\right)\chi_\mp = 0\,, \tag{3.11}$$

[5] This assumption is valid generically, but for the special case of the adS vacuum there will be additional Killing spinors for which this assumption is not valid; their existence ensures that all supersymmetries are preserved in this adS vacuum.

and hence we deduce that

$$\dot{\phi} = 0\,, \qquad \dot{\sigma} = \mp 8gW'\,. \tag{3.12}$$

We have now confirmed both that $\dot{\phi} = 0$ and the first-order equations (3.3).

Returning now to the Killing spinor equation, we note that $A_r = 0$ for $\dot{\phi} = 0$, so the component of this equation perpendicular to the wall is simply

$$\dot{\chi}_{\mp} = \pm gW\chi_{\mp}\,. \tag{3.13}$$

It follows that the Killing spinors take the form

$$\chi = \mathrm{e}^{A/2}\xi_{\mp}\,, \qquad \Gamma\,\xi_{\mp} = \mp\xi_{\mp}\,, \tag{3.14}$$

for constant real spinor $\xi_{\mp}$. Note that because Γ is real, the reality of $\xi_{\mp}$ is consistent with it being an eigenspinor of Γ.

For the top (bottom) sign in (3.3) there is a solution only for $r > 0$ ($r < 0$). Let us choose the top sign, corresponding to $r > 0$. Positivity of $|z| \equiv \rho$ requires $\sigma > 0$, and the solution compatible with this requirement is

$$\sigma = 2\log\coth\left(gr\right)\,, \tag{3.15}$$

and hence

$$\rho \equiv \tanh\left(\sigma/2\right) = \mathrm{sech}\left(2gr\right)\,, \qquad \cosh\left(\sigma/2\right) = \coth\left(2gr\right)\,. \tag{3.16}$$

The equation for A (again for the top sign) is now easily solved, and the solution is

$$A = \log\sinh\left(2gr\right)\,. \tag{3.17}$$

The domain wall metric is therefore

$$ds^2 = dr^2 + \sinh^2\left(2gr\right)ds^2(\mathrm{Mink3})\,. \tag{3.18}$$

This metric is singular at $r = 0$ but is asymptotic to adS as $r \to \infty$. We therefore have a solution that is defined for $r > 0$ and is asymptotic to the adS vacuum with $\sigma \equiv 0$ as $r \to \infty$. The Killing spinors for this solution are

$$\chi = \left[\sinh\left(2gr\right)\right]^{\frac{1}{2}}\xi_-\,, \qquad \tau_1\Gamma_r\,\xi_- = -\xi_-\,, \tag{3.19}$$

which shows that the domain-wall solution is half-supersymmetric. As noted earlier, our model is an $N = 2$ truncation of SO(8) gauged $N = 8$ supergravity, and any solution can be lifted to a solution of 11-dimensional supergravity. The domain wall solution found here can be shown to be equivalent to one found in [27], where the lift to $D = 11$ was interpreted as a continuous distribution of M2-branes.

3.2 Cosmologies

As observed in [9], we may interpret the first-order equations (3.3) as equations determining a pseudo-supersymmetric cosmology; in this case g is imaginary and we may choose

$$g = \mathrm{i}|g| \,. \tag{3.20}$$

The cosmological counterpart of (3.4) is then found to be

$$\left(\dot{A} + 2|g|W\tilde{\Gamma}\right)\chi = 0\,, \qquad \tilde{\Gamma} = \mathrm{i}\Gamma_0\tau_1\,. \tag{3.21}$$

If we now write $\chi = \chi_+ + \chi_-$ as in the domain-wall case, but now with $\tilde{\Gamma}\chi_\pm = \pm\chi_\pm$ then we find as before that

$$\dot{A} = \pm 2|g|W\,, \qquad \chi = \chi_\mp\,. \tag{3.22}$$

The 'photino equation' then leads, as before, to

$$\dot{\phi} = 0\,, \qquad \dot{\sigma} = \mp 8|g|W'\,. \tag{3.23}$$

We have now confirmed both that $\dot{\phi} = 0$ and (3.3) for the cosmology case. The associated pseudo-Killing spinors are given by

$$\chi = e^{A/2}\xi_\mp\,, \qquad \tilde{\Gamma}\,\xi_\mp = \mp\xi_\mp\,. \tag{3.24}$$

Although $\tilde{\Gamma}$ is imaginary, it anticommutes with τ_3, so the projection onto an eigenspace of $\tilde{\Gamma}$ is compatible with the twisted reality condition

$$\xi_\mp^* = \tau_3\,\xi_\mp\,. \tag{3.25}$$

Choosing the top sign in (3.3), which now corresponds to a cosmological solution with $t > 0$, we find that the pseudo-supersymmetric solution has

$$\sigma = 2\log\coth\left(|g|t\right)\,, \tag{3.26}$$

and a metric

$$ds^2 = -dt^2 + \sinh^2\left(2|g|t\right)dl^2(\mathrm{E}3)\,. \tag{3.27}$$

There is a big bang singularity at $t = 0$ after which we have an expanding universe that approaches the pseudo-supersymmetric dS vacuum as $t \to \infty$. The (time-dependent) pseudo-Killing spinors for this solution are

$$\chi(t) = \left[\sinh\left(2|g|t\right)\right]^{\frac{1}{2}}\xi_-\,, \qquad \mathrm{i}\Gamma_0\tau_1\,\xi_- = -\xi_-\,, \tag{3.28}$$

where ξ_- is a constant spinor subject to the twisted reality condition (3.25).

Just as our 'standard' supergravity model was the $N = 2$ truncation of an S^7 compactification of $D = 11$ supergravity, our pseudo-supergravity model is an $N = 2$ truncation of an adS_7 'compactification' of the (two-time) M*-theory [30]. It follows that the cosmological solution lifts to a (presumably supersymmetric) solution of M*-theory, and it is possible that its big bang singularity could be resolved in this context.

4 Discussion

As originally conceived, pseudo-supersymmetry [9,11] was merely a property of certain cosmological solutions, unrelated to any symmetry. In view of the results of this paper we should perhaps refer to this original concept as 'fake' pseudo-supersymmetry, because we have seen that it is possible to view it as arising, in special cases, as a consequence of a local 'supersymmetry' of an underlying 'pseudo-supergravity' theory in much the same way as 'fake supersymmetry' arises (again in special cases) as a consequence of the local supersymmetry of some supergravity theory. Pseudo-supergravity theories have vector ghosts, and are therefore non-unitary, but their existence is still a non-trivial mathematical fact, and the ghost sector plays no role in our analysis.

The concept of pseudo-supersymmetric cosmology arose from an application to fake supergravity of the domain-wall/cosmology (DW/C) correspondence, which relates domain wall solutions of a model with a scalar potential V to cosmological solutions of the same model but with scalar potential $-V$ [9,12]. As a consequence one may view a given model with potential V as the 'dual' of the same model with potential $-V$. This extension of the correspondence from solutions to models is trivial in the 'fake' setting but non-trivial in the supergravity setting. We have shown that a particular U(1) gauged $N = 2$, $D = 4$ Maxwell-Einstein supergravity with an adS vacuum has a 'dual' pseudo-supergravity theory with a dS vacuum, found by imposing 'twisted reality' conditions on the fermions. The two models are dual in the sense that not only are the bosonic truncations the same up to the flip of sign of the scalar potential, but also in the sense that a supersymmetric domain wall of the standard supergravity theory is dual, in the sense of the DW/C correspondence, to a supersymmetric cosmology of the pseudo-supergravity theory. The (a)dS vacua of these theories are special cases of this correspondence, with enhanced supersymmetry in both cases.

One motivation for this work was to understand the implications of pseudo-supersymmetry. One such implication was recently presented in [31], where it was shown that scaling solutions

that are pseudo-supersymmetric must describe geodesic curves in target space. Another possible implication concerns stability in de Sitter space: the DW/C correspondence maps the well-known Breitenlohner-Freedman (BF) stability bound on scalar field masses in adS space to an upper bound on scalar field masses in dS space, such that dS vacua can be pseudo-supersymmetric only if all scalar field masses satisfy the bound [11]. It may be verified that this 'cosmological' bound *is* satisfied by the supersymmetric dS vacuum of the pseudo-supergravity theories considered here; this is just a consequence of the fact that the BF bound is satisfied by the supersymmetric adS vacuum of the 'standard' theory. An obvious question is whether the cosmological version of the BF bound may also be viewed as a stability bound. In this context we note that results of a recent article [32] suggest that particles with masses above the bound are indeed unstable, although FLRW spacetimes are known to be classically stable for any mass; see [33] for recent rigorous analysis of this issue. In any case, the cosmological bound has a simple group theoretic meaning when phrased in terms of particles created by scalar fields, where a 'particle' in dS space is identified with a unitary irreducible representation (UIR) of the de Sitter group. These UIRs are classified into principal, complementary and discrete series. Particles with mass above the bound correspond to the principal series while particles with positive mass below the bound correspond to the complementary series. It is a group theoretic fact that there are no unitary fermionic complementary series [34], so that particles with a non-zero mass below the bound cannot be paired with fermions by any symmetry acting on a positive-definite Hilbert space. This shows that if pseudo-supersymmetry is to be realized as a symmetry of a dS vacuum then the Hilbert space on which this symmetry acts must be indefinite, as indeed it is for the dS vacua of pseudo-supergravity theories because of the vector ghosts.

Normal supersymmetry has been instrumental in many recent advances by providing a means of obtaining otherwise inaccessible exact results, and pseudo-supersymmetry could play a similarly important role in cosmology, e.g. in the context of a dS/CFT correspondence. Our realization of the DW/C correspondence in supergravity shows that pseudo-supersymmetry is not an 'accidental' property of cosmology but one that is related to a genuine, and mathematically non-trivial, symmetry.

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References

[1] D. Z. Freedman, C. Núñez, M. Schnabl and K. Skenderis, “Fake supergravity and domain wall stability”, Phys. Rev. D **69**, 104027 (2004) [arXiv:hep-th/0312055].

[2] K. Skenderis and P. K. Townsend, “Gravitational stability and renormalization-group flow,” Phys. Lett. B **468**, 46 (1999) [arXiv:hep-th/9909070].

[3] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, “Modeling the fifth dimension with scalars and gravity,” Phys. Rev. D **62**, 046008 (2000) [arXiv:hep-th/9909134].

[4] A. Celi, A. Ceresole, G. Dall’Agata, A. Van Proeyen and M. Zagermann, “On the fakeness of fake supergravity”, Phys. Rev. D **71** (2005) 045009 [arXiv:hep-th/0410126].

[5] M. Zagermann, “N = 4 fake supergravity,” Phys. Rev. D **71** (2005) 125007 [arXiv:hep-th/0412081].

[6] J. Sonner and P. K. Townsend, “Axion-Dilaton domain walls and fake supergravity,” Class. Quant. Grav. **24** (2007) 3479 [arXiv:hep-th/0703276].

[7] H. Elvang, D. Z. Freedman and H. Liu, “From fake supergravity to superstars,” arXiv:hep-th/0703201.

[8] J. Sonner and P. K. Townsend, “Dilaton domain walls and dynamical systems”, Class. Quant. Grav. **23**, 441 (2006) [arXiv:hep-th/0510115].

[9] K. Skenderis and P. K. Townsend, “Hidden supersymmetry of domain walls and cosmologies,” Phys. Rev. Lett. **96**, 191301 (2006) [arXiv:hep-th/0602260];

[10] K. Skenderis and P. K. Townsend, "Hamilton-Jacobi method for domain walls and cosmologies," Phys. Rev. D **74**, 125008 (2006) [arXiv:hep-th/0609056].

[11] K. Skenderis and P. K. Townsend, "Pseudo-supersymmetry and the domain-wall / cosmology correspondence," J. Phys. A: Math. Theor. **40** (2007) 6733-6741 [arXiv:hep-th/0610253].

[12] M. Cvetic and H. H. Soleng, "Naked singularities in dilatonic domain wall space times," Phys. Rev. D **51** (1995) 5768 [arXiv:hep-th/9411170].

[13] W. Nahm, "Supersymmetries and their representations," Nucl. Phys. B **135** (1978) 149.

[14] J. Lukierski and A. Nowicki, "Supersymmetry in the presence of positive cosmological constant", preprint Wrocław, no. 609, March 1984

[15] K. Pilch, P. van Nieuwenhuizen and M. F. Sohnius, "De Sitter superalgebras and supergravity," Commun. Math. Phys. **98** (1985) 105.

[16] J. Lukierski and A. Nowicki, "All possible de Sitter superalgebras and the presence of ghosts," Phys. Lett. B **151** (1985) 382.

[17] B. de Wit and A. Zwartkruis, "SU(2,2/1,1) supergravity and $N = 2$ supersymmetry with arbitrary cosmological constant", Class. Quant. Grav. **4** (1987) L59.

[18] R. D'Auria and S. Vaulà, "$D = 6$, $N = 2$, $F(4)$-supergravity with supersymmetric de Sitter background," JHEP **0209** (2002) 057 [arXiv:hep-th/0203074].

[19] E. A. Bergshoeff, J. Hartong, A. Ploegh, J. Rosseel and D. Van den Bleeken, "Pseudo-supersymmetry and a tale of alternate Realities," arXiv:0704.3559 [hep-th].

[20] B. de Wit and A. Van Proeyen, "Potentials and symmetries of general gauged $N = 2$ supergravity – Yang-Mills models", Nucl. Phys. **B245** (1984) 89

[21] A. Van Proeyen, "Tools for supersymmetry", Annals of the University of Craiova, Physics AUC **9 (part I)** (1999) 1–48, `hep-th/9910030`

[22] S. Cecotti, L. Girardello and M. Porrati, "Ward identities of local supersymmetry and spontaneous breaking of extended supergravity," in *New trends in particle theory*, proc. of the 9th Johns Hopkins Workshop, Firenze, World scientific, 1985, ed. L. Lusanna

[23] K. Behrndt and M. Cvetič, "Anti-de Sitter vacua of gauged supergravities with 8 supercharges," Phys. Rev. D **61** (2000) 101901 [arXiv:hep-th/0001159].

[24] S. Ferrara and L. Maiani, "An introduction to supersymmetry breaking in extended supergravity," in *Relativity, supersymmetry and cosmology*, proc. of SILARG V, 5th Latin American Symp. on Relativity and Gravitation, Bariloche, Argentina, Jan 1985, World Scientific, ed. O. Bressan, M. Castagnino, V.H. Hamity

[25] S. Cecotti, L. Girardello and M. Porrati, "Constraints on partial superhiggs," Nucl. Phys. B **268** (1986) 295.

[26] M. J. Duff and J. T. Liu, "Anti-de Sitter black holes in gauged $N = 8$ supergravity," Nucl. Phys. B **554**, 237 (1999) [arXiv:hep-th/9901149].

[27] M. Cvetič, S. S. Gubser, H. Lü and C. N. Pope, "Symmetric potentials of gauged supergravities in diverse dimensions and Coulomb branch of gauge theories," Phys. Rev. D **62**, 086003 (2000) [arXiv:hep-th/9909121].

[28] M. Cvetič, H. Lü and C. N. Pope, "Four-dimensional $N = 4$, SO(4) gauged supergravity from $D = 11$," Nucl. Phys. B **574** (2000) 761 [arXiv:hep-th/9910252].

[29] E. Bergshoeff and A. Van Proeyen, "The many faces of OSp(1|32)", Class. Quant. Grav. **17** (2000) 3277–3304, `hep-th/0003261`

[30] J. T. Liu, W. A. Sabra and W. Y. Wen, "Consistent reductions of IIB*/M* theory and de Sitter supergravity," JHEP **0401**, 007 (2004) [arXiv:hep-th/0304253].

[31] W. Chemissany, A. Ploegh and T. Van Riet, "A note on scaling cosmologies, geodesic motion and pseudo-susy," arXiv:0704.1653 [hep-th].

[32] J. Bros, H. Epstein and U. Moschella, "Lifetime of a massive particle in a de Sitter universe," arXiv:hep-th/0612184.

[33] H. Ringström, "Future stability of the Einstein-Non-Linear scalar field system", preprint.

[34] R. Takahashi, "Sur les représentations unitaires des groupes de Lorentz généralisés", Bulletin de la Société Mathématique de France, 91 (1963), p. 289-433